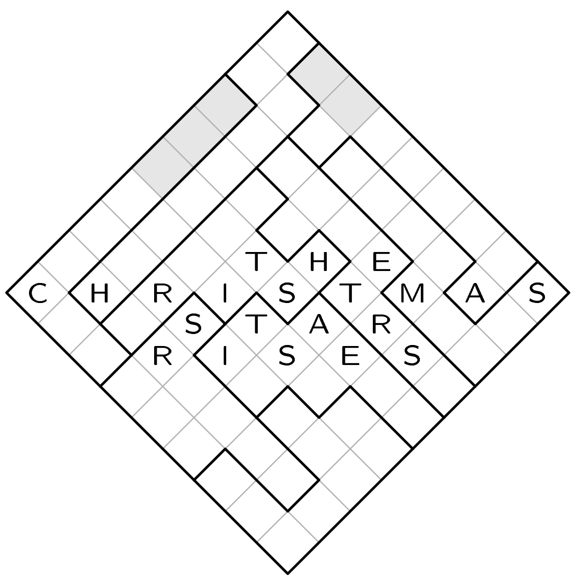
Chapter 1

Sudoku For Christmas

Here's a type of "jigsaw sudoku" puzzle that contains a holiday message:



This 9×9 diamond contains 9 downward diagonals, 9 upward diagonals, and 9 bordered regions. Some of the boxes are already filled in to say THE CHRISTMAS STAR RISES. Notice that nine letters are used: C, H, A, R, M, S, I, T, E.

Fill in the other boxes so that every diagonal and every region contains C, H, A, R, M, S, I, T, E in some order. (There's only one way.)

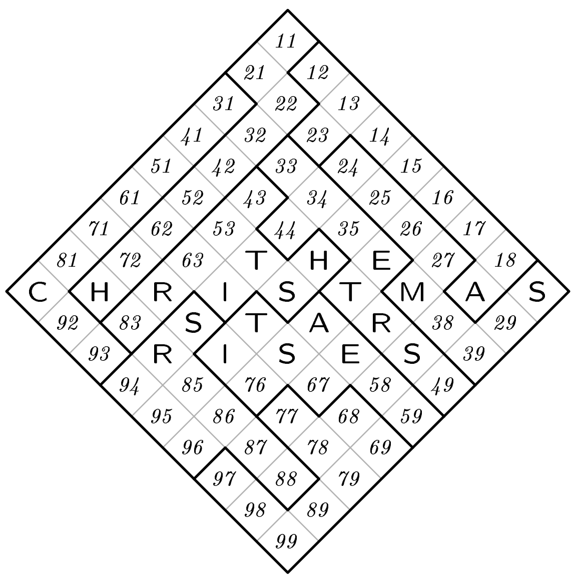
Hint: Where can you put the letter A into the top northwest-to-southeast diagonal row, without a conflict?

When you've solved the puzzle, a secret word will appear in the five lightly-shaded boxes near the top. Have fun!

Hints on solving THE CHRISTMAS STAR RISES

Warning: It's best to uncover only one step of this solution at a time, so as not to spoil the "aha"-type fun of discovery. Read further *only* when you're stuck; then you'll soon find your skills rising like a star!

Let's put labels into the blank cells so that we can talk about them:



It will be convenient to refer to a downward diagonal like *11*, *12*, ... as "row 1," and to an upward diagonal like ..., *21*, *11* as "column 1."

Since many of the cells are filled in, we have only a limited number of ways to put letters into the unknown places. For example, look at cell *35*, and consider the possibilities in turn: Should *35* be filled with

C, H, A, R, M, S, I, T, or E?

Only C and A are not blocked. The reason is that row 3 already contains E and M; column 5 already contains H, S, T, I; and the region around cell *35* already contains E, T, R, S.

Now notice that *11* is forced to be A, because there must an A in row 1 (and A can't go into *12*, *13*, ..., *18*, because that region already contains its A).

Setting *11* = A, we can conclude in a similar way that *92* must also be A. (There must be an A in column 2.) Get it?

Next, notice that *39* is also forced to be A — yes, we're on an "A binge" — because there must be an A in the region that contains cells *24*, *25*, *26*, *27*, *38*, *39*, *29*. The A doesn't fit anywhere else in that region.

Aha! With *39* = A, only one possibility remains for cell *35*, because we've already noted that *35* has to be either A or C.

⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆

Let's pause at this point to review the methods that we've used. There are four basic types of forcing steps that make progress in this puzzle, and we've now seen examples of each.

Row move: There's just one way to put a certain letter into a row.

Column move: There's just one way to put a letter into a column.

Region move: There's just one way to put a letter into a region.

Position move: There's just one letter that can go into a certain cell.

First we used a row move to conclude that *11* = A.
Then we used a column move to conclude that *92* = A.
Then we used a region move to conclude that *39* = A.
And we used a position move to conclude that *35* = C.

⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆

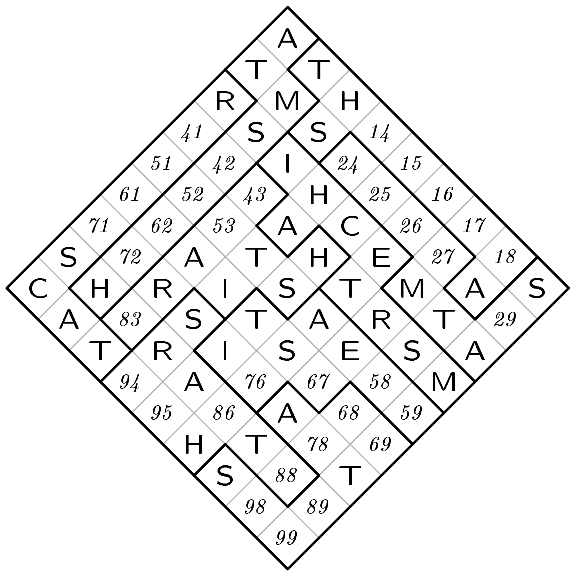
Armed with these four tools, and with a good eye (or a good system of organized notes) to see when they apply, we can in fact fill in 25 of the unknown cells without needing to erase anything!

Here is one scenario by which all forced moves can be successively discovered, not necessarily in this order — but remember that it's best not to peek at them until you've given this exercise the old college try:

11 = A (row)	*33* = I (pos)	*93* = T (col)
92 = A (col)	*49* = M (pos)	*31* = R (pos)
39 = A (reg)	*44* = A (pos)	*38* = T (pos)
35 = C (pos)	*85* = A (col)	*21* = T (col)
23 = S (reg)	*63* = A (col)	*22* = M (row)
32 = S (reg)	*77* = A (row)	*12* = T (col)
81 = S (col)	*96* = H (reg)	*97* = S (col)
34 = H (pos)	*13* = H (col)	*79* = T (row)

and finally *87* = T (col).

So we've progressed to the following partial solution:



At this point nothing is forced, and we must resort to guessing.

(Well, we do know that the secret word "*51 41 31 12 13*" is __RTH; in the Christmas spirit we know therefore that *51* = M and *41* = I. But let's suppose that no secret word clue had been given.)

The guessing strategy that will be described in the rest of these notes might not be the best way to proceed, but it does work. The main idea is to find a row, column, region, or cell for which there are only a small number of cases to try. For example, only two choices remain for *17*: That cell must be either C or I.

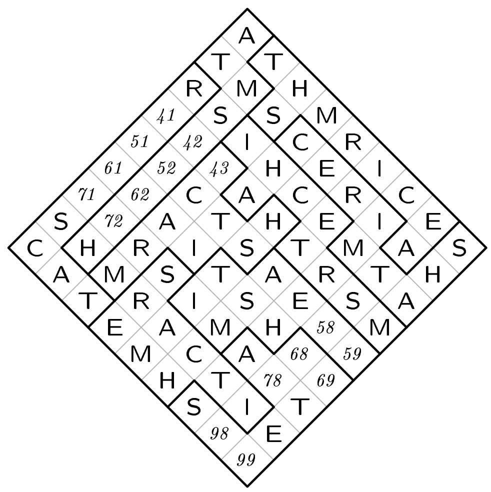
Let's try *17* = C. Then more things are forced, namely

67 = H (pos)	*95* = M (pos)	*86* = C (pos)
27 = I (pos)	*15* = R (pos)	*76* = M (pos)
29 = H (row)	*94* = E (pos)	*88* = I (pos)
24 = C (col)	*14* = M (pos)	*89* = E (pos)
26 = R (pos)	*16* = I (pos)	*83* = M (pos)
25 = E (pos)	*18* = E (pos)	*53* = C (pos)

and we arrive at the partial solution at the top of the next page.

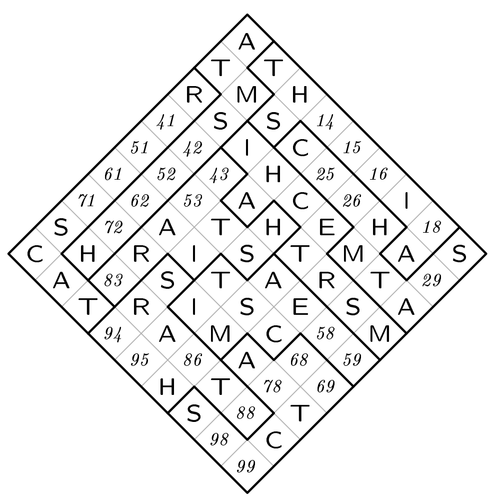
Oops: We're stuck, because there's no way to put C into the region that contains *58* and *59*. So we made a bad guess, and we have to backtrack. Since *17* cannot be C, we've discovered in fact that *17* = I.

OK, we must reluctantly part with the diagram



and return to our previous state, except that we are now allowed to set $\mathit{17} = \mathsf{I}$. Unfortunately this new fact doesn't force any more moves; we'll have to make yet another guess.

Well, $\mathit{24}$ has to be either C or E. Suppose $\mathit{24} = \mathsf{C}$; then $\mathit{27} = \mathsf{H}$ (pos); $\mathit{67} = \mathsf{C}$ (pos); $\mathit{76} = \mathsf{M}$ (pos); $\mathit{89} = \mathsf{C}$ (col); and we've reached another impasse:



There's no way to put C into the region that contains $\mathit{88}$. But hey, we have deduced that $\mathit{24} = \mathsf{E}$.

And now it turns out that we're home free. Everything else is forced:

25 = R (pos)	*41* = I (pos)	*58* = H (pos)
94 = M (pos)	*42* = C (pos)	*67* = C (pos)
14 = C (pos)	*72* = E (pos)	*27* = H (pos)
95 = E (pos)	*62* = R (pos)	*68* = M (pos)
15 = M (pos)	*52* = I (pos)	*88* = I (pos)
16 = R (pos)	*71* = H (pos)	*86* = C (pos)
18 = E (pos)	*51* = M (pos)	*26* = I (pos)
76 = M (col)	*61* = E (pos)	*29* = C (pos)
83 = M (row)	*78* = C (pos)	*89* = E (pos)
53 = C (pos)	*69* = H (pos)	*98* = R (pos)
43 = E (pos)	*59* = R (pos)	*99* = I (pos)

We needn't reproduce the final diagram here.

Merry Christmas and Happy New Year!

Reference

[1] Bob Harris, "The Grand Time Sudoku and the law of leftovers," *Mathematical Wizardry for a Gardner* (Wellesley, Massachusetts: A K Peters, 2009), 55–57.