SOME APPLICATIONS OF LOGICAL SYNTAX TO DIGITAL COMPUTER PROGRAMMING

A thesis presented

by

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in the subject of

Applied Mathematics

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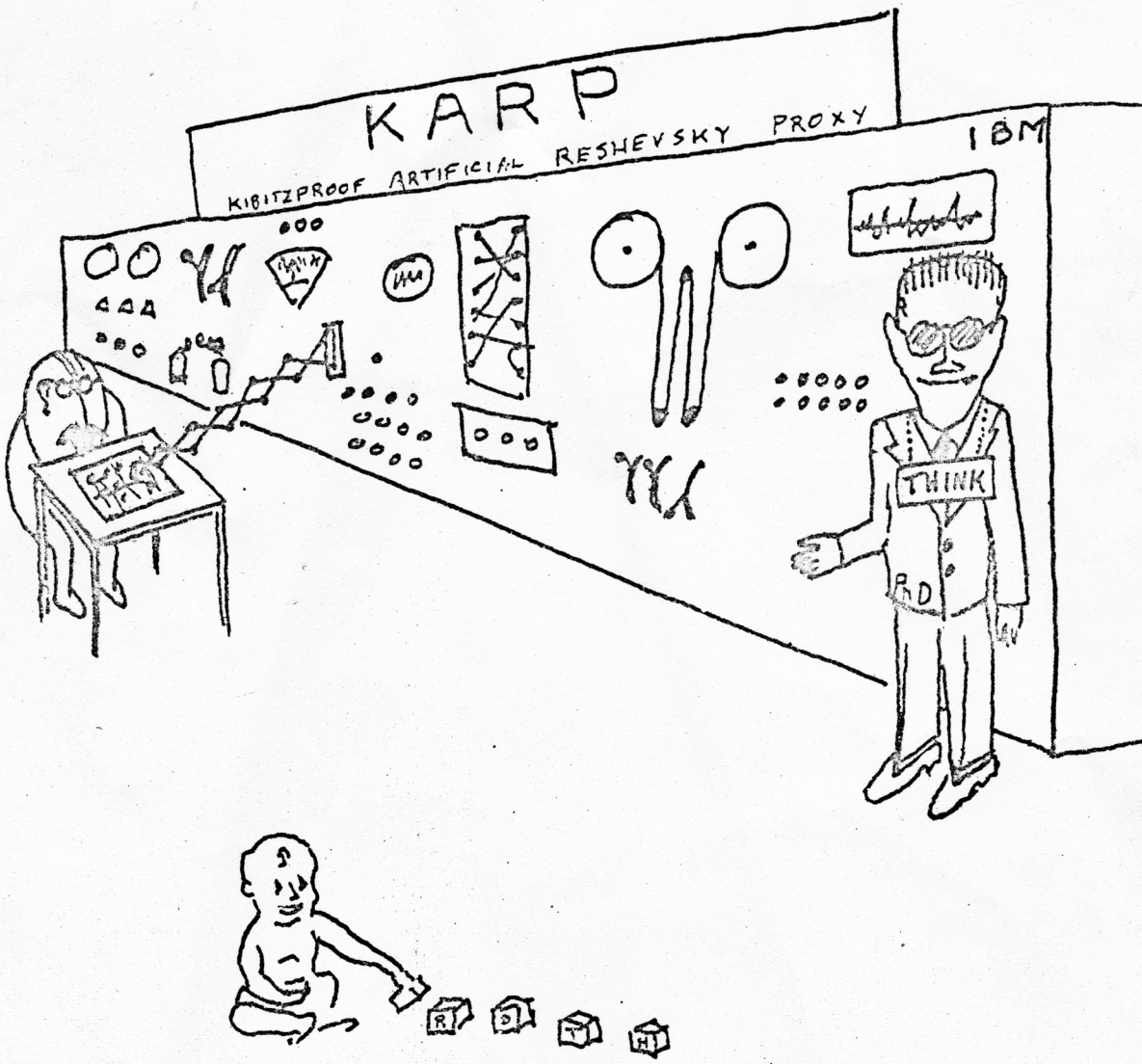
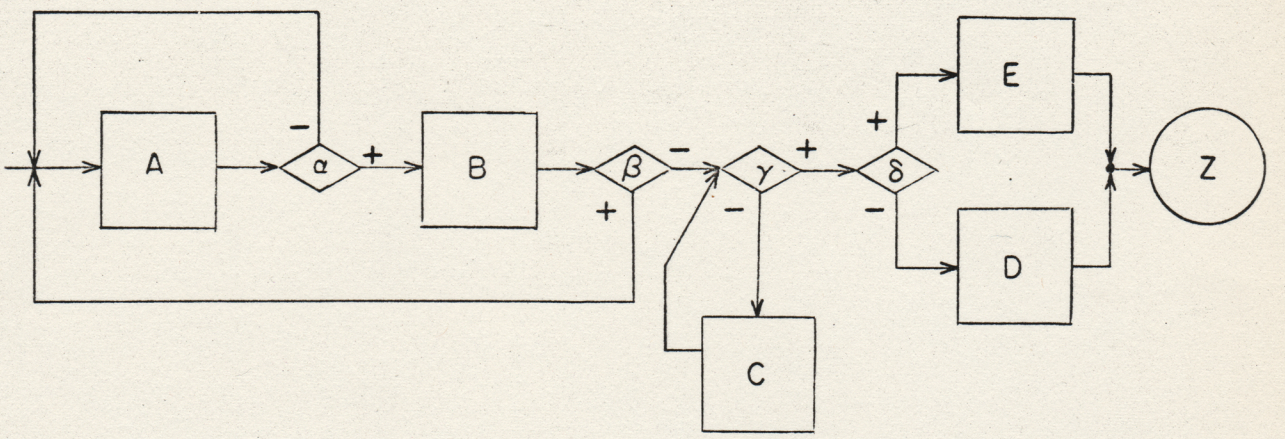
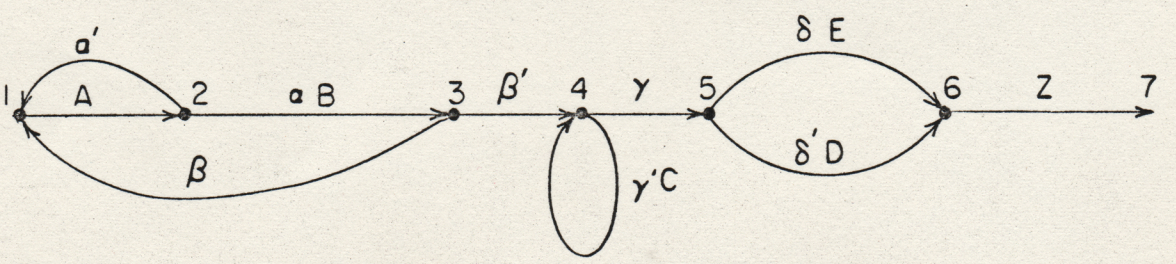




A Schematic Flowchart

Fig. 2-1

Step 2 consists of replacing every series chain $A_1, A_2, \ldots, A_n$, by a single arc labelled with the name $A_1 A_2 \cdots A_n$. When Step 2 is performed on the graph of Fig. 2-3, the graph of Fig. 2-4 results. In the graph of



Graph Obtained in Step 2

Fig. 2-4

The connection matrix of Fig. 2-5 gives a complete topological description both of the flowchart of Fig. 2-1 and of the topological graph of Fig. 2-4. In practice, the topological graph need not be drawn, since

$$\begin{pmatrix}
0 & A & 0 & 0 & 0 & 0 & 0 \\
\alpha' & 0 & \alpha B & 0 & 0 & 0 & 0 \\
\beta & 0 & 0 & \beta' & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma' C & \gamma & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \delta E + \delta' D & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Z \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

A Connection Matrix

Fig. 2-5

1. Oettinger, A. G., "A New Basic Approach to Automatic Data Processing," Report No. 3 by the Staff of the Computation Laboratory to the Electronic Research Steering Committee of the A.G.A. and E.E.I., Harvard Computation Laboratory, August, 1956.

2. Karp, R. M., "The Assembly of Address-free Programs for Automatic Bill Calculation," Report No. 5 by the Staff of the Computation Laboratory to the Electronic Research Steering Committee of the A.G.A. and E.E.I., Harvard Computation Laboratory, May, 1957.

3. Newman, E. A. and Wright, M. A., "An Automatic Floating-Address Machine," Proceedings of the Institute of Electrical Engineers, Vol. 103, supplement no. 1, 1956.

19. Gorn, S., "Standardized Programming Methods and Universal Coding," Journal of the Association for Computing Machinery, 4, No. 3, July, 1957, pp. 254-273.

20. Karp, R. M., "Methods of Evaluating Cage and Fortran for Use in Specific Applications," General Electric Co., Aircraft Nuclear Propulsion Dept., September, 1957.

21. Kleene, S. C., Introduction to Metamathematics, Van Nostrand, New York, 1952.

```
000   11 48031 480        030   C  057------
001   31 54031 600        031   B  056C  055
002   30 660B  053        032   B  057C  056
003   C  054K             033   ------U  019
004  [B  480T1 007]       034   ----------
005  [S- 480J  055]       035   ----------
006   ------U  008        036  [B  056C  720]
007   C  055------        037   B  054L  058
008   K     C  002        038   ------Q8 040
009  [B  600T2 012]       039   ------U  043
010  [S- 600J  056]       040   52 72052 780
011   ------U  013        041   61    62
012   ------C  056        042   ------999999
013   B  056L  055        043   B  004A- 059
014   ------T3 016        044   C  004B  005
015   ------U  019        045   A- 059C  005
016   B  055C  057        046   B  009A- 059
017   B  056C  055        047   C  009B  010
018   B  057C  056        048   A- 059C  010
019   L  056B  053        049   B  036A- 053
020   ------Q4 036        050   C  036B  054
021   D- 053J  001        051   A- 053C  054
022   L  001P- 055        052   ------U  004
023   06    C  057        053   ----------1
024   B  05605            054   ----------
025   K     M- 057        055   ----------
026   S- 055;1            056   ----------
027   .1    L  056        057   ----------
028   ------Q5 036        058   000000000100
029   L  002Q6 036        059   000001000000
```

A Program for Executing the Euclidean Algorithm

Fig. 3-4

```
math
~/tmp[1]> Mathematica 9.0 for Linux x86 (64-bit)
Copyright 1988-2013 Wolfram Research, Inc.

In[1]:= gcd019[big_,sm_,time_]:=Block[{},trace[{a,big,sm,time}];
                          If[time>100,Loop,
                           If[sm==1,1,
                            If[sm==0,Overflow,
                             gcd021[big,sm,time]]]]]

In[2]:= gcd021[big_,sm_,time_]:=Block[{t,rA,t1,rL,rem},
                            t=Floor[10^11/sm];
                             rA=Floor[big*t/10^11];
                              t1=Mod[10^6rA,10^11];
                               rL=Mod[10^5sm,10^11];
                                rem=Abs[rL*t1/10^11-big];
                            trace[{b,t,rA,t1,rL,rem,time}];
                            If[rem==big || rem==0,sm,gcd019[sm,rem,time+1]]]

In[3]:= gcd[x_,y_]:=If[Abs[y]>Abs[x],gcd019[Abs[y],Abs[x],0],gcd019[Abs[x],Abs[y],0]]

In[4]:= trace[l_]:=Print[l]

In[5]:= gcd[314159,271828]
{a, 314159, 271828, 0}
{b, 367879, 1, 1000000, 27182800000, 42331, 0}
{a, 271828, 42331, 1}
{b, 2362334, 6, 6000000, 4233100000, 17842, 1}
{a, 42331, 17842, 2}
{b, 5604752, 2, 2000000, 1784200000, 6647, 2}
{a, 17842, 6647, 3}
{b, 15044380, 2, 2000000, 664700000, 4548, 3}
{a, 6647, 4548, 4}
{b, 21987686, 1, 1000000, 454800000, 2099, 4}
{a, 4548, 2099, 5}
{b, 47641734, 2, 2000000, 209900000, 350, 5}
{a, 2099, 350, 6}
{b, 285714285, 5, 5000000, 35000000, 349, 6}
{a, 350, 349, 7}
{b, 286532951, 1, 1000000, 34900000, 1, 7}
{a, 349, 1, 8}

Out[5]= 1
```

```
In[6]:= gcd[37 31415, 37 2718]
{a, 1162355, 100566, 0}
{b, 994371, 11, 11000000, 10056600000, 56129, 0}
{a, 100566, 56129, 1}
{b, 1781610, 1, 1000000, 5612900000, 44437, 1}
{a, 56129, 44437, 2}
{b, 2250376, 1, 1000000, 4443700000, 11692, 2}
{a, 44437, 11692, 3}
{b, 8552856, 3, 3000000, 1169200000, 9361, 3}
{a, 11692, 9361, 4}
{b, 10682619, 1, 1000000, 936100000, 2331, 4}
{a, 9361, 2331, 5}
{b, 42900042, 4, 4000000, 233100000, 37, 5}
{a, 2331, 37, 6}
{b, 2702702702, 62, 62000000, 3700000, 37, 6}
{a, 37, 37, 7}
{b, 2702702702, 0, 0, 3700000, 37, 7}

Out[6]= 37

In[7]:= gcd[37 314159, 37 271828]
{a, 11623883, 10057636, 0}
{b, 9942, 1, 1000000, 5763600000, 11566247, 0}
{a, 10057636, 11566247, 1}
{b, 8645, 0, 0, 56624700000, 10057636, 1}

Out[7]= 11566247

In[8]:= gcd[0,7]
{a, 7, 0, 0}

Out[8]= Overflow

In[9]:= gcd[200004,2]
{a, 200004, 2, 0}
{b, 50000000000, 100002, 2000000, 200000, 200000, 0}
{a, 2, 200000, 1}
{b, 500000, 0, 0, 20000000000, 2, 1}

Out[9]= 200000

In[10]:= trace[l_]:=Null

In[11]:= Do[If[gcd[x,y]!=GCD[x,y],Print[{x,y}]],{x,200004},{y,x-1}]
{200004, 2}

In[12]:= Quit
```

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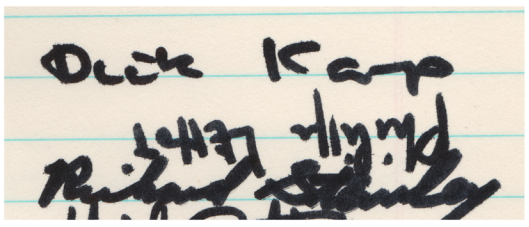

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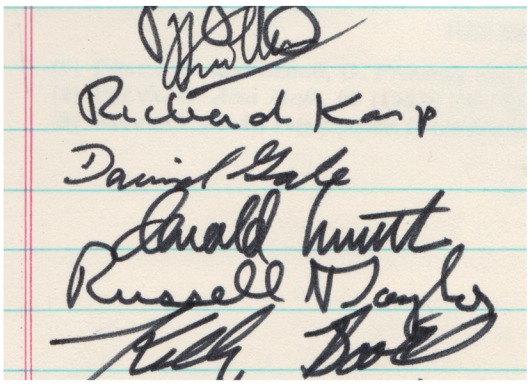

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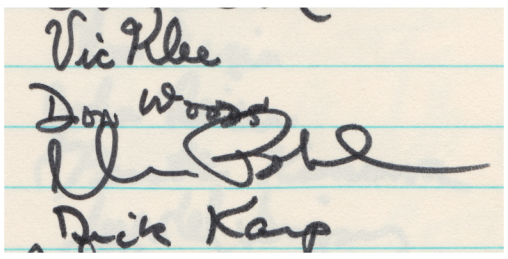

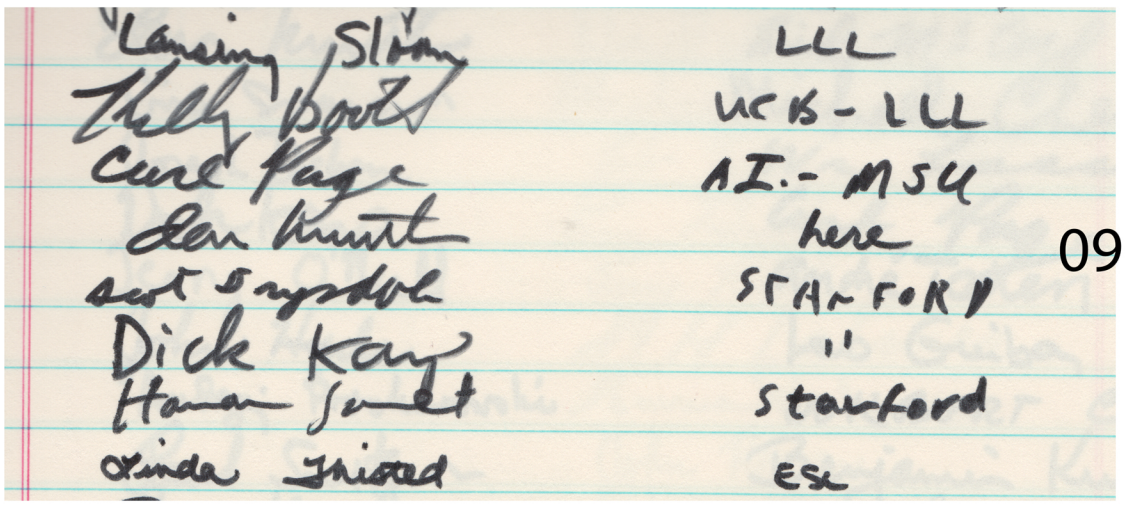

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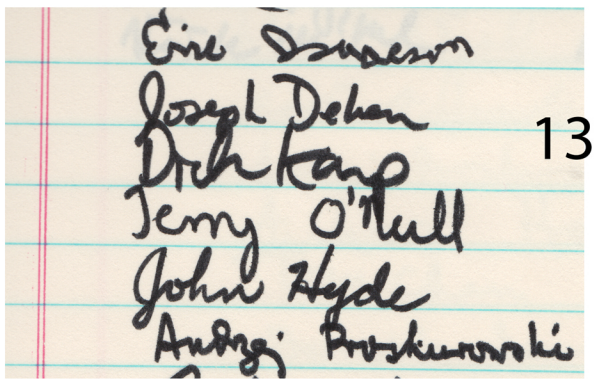
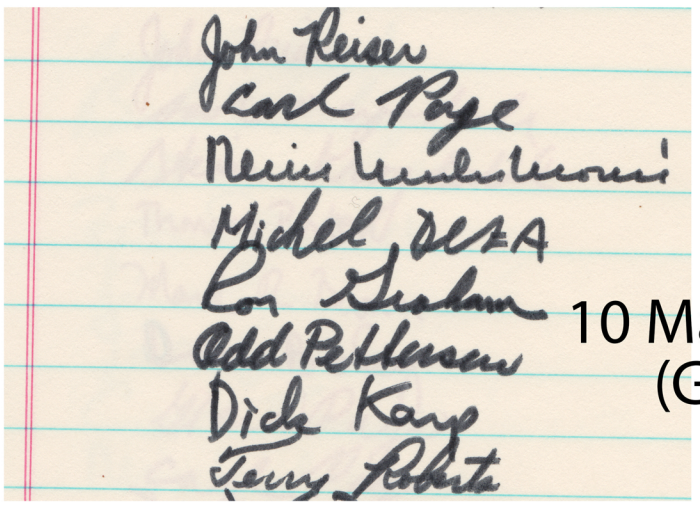
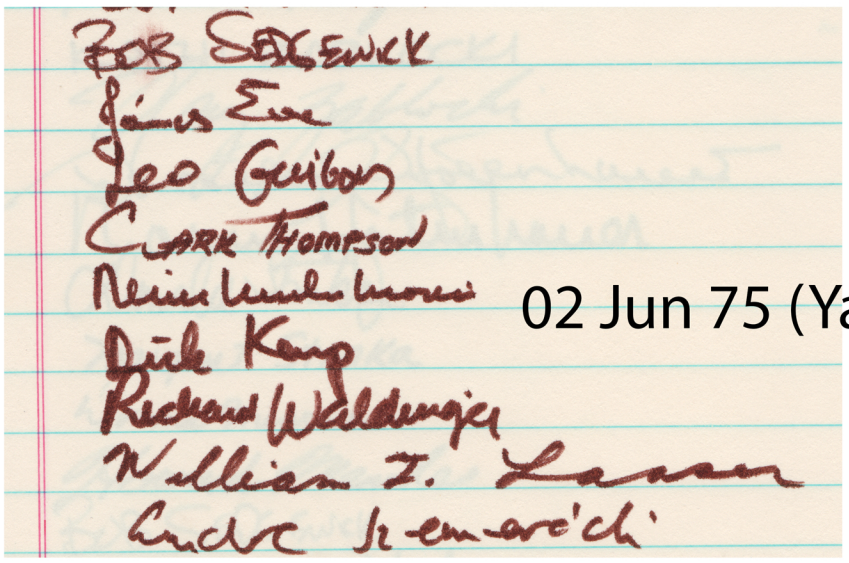


13 Jan 75 (Diaconis)



10 Mar 75 (Garey & Johnson)



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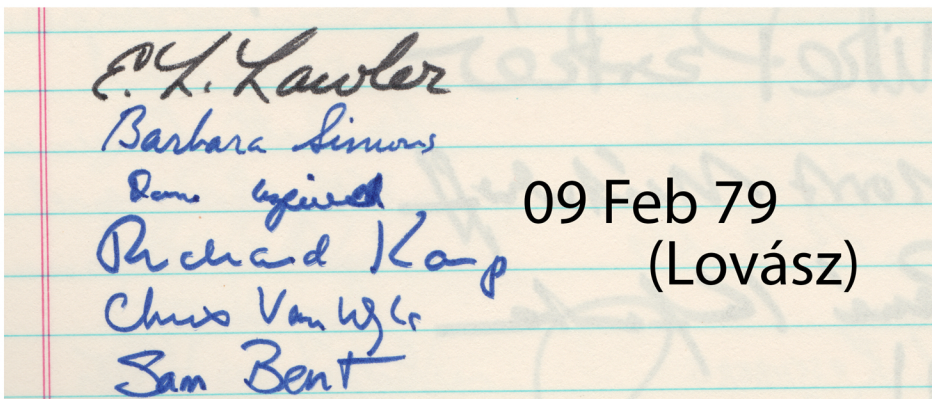

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